LOCAL NON-BOSSINESS AND PREFERENCES OVER COLLEAGUES

EDUARDO DUQUE, JUAN S. PEREYRA, AND JUAN PABLO TORRES-MARTÍNEZ

ABSTRACT. The student-optimal stable mechanism (DA), the most popular mechanism in school choice, is the only one that is both stable and strategy-proof. However, when DA is implemented, a student can change the schools of others without changing her own. We show that this drawback is limited: a student cannot change her classmates without modifying her school. We refer to this new property as *local non-bossiness*. Along with strategy-proofness, it ensures a local notion of group strategy-proofness in which manipulating coalitions are restricted to students in the same school. Furthermore, local non-bossiness plays a crucial role in incentives when students have preferences over their colleagues. As long as students first consider the school to which they are assigned and then their classmates, DA induces the only stable and strategy-proof mechanism in this preference domain. To some extent, this is the maximal domain in which a stable and strategy-proof mechanism exists for any school choice context.

KEYWORDS: School Choice - Preferences over Colleagues - Local Non-bossiness
JEL CLASSIFICATION: D47, C78.

1. INTRODUCTION

In the last two decades, an increasing number of centralized school admission systems have been implemented worldwide.[1] In this context, the *student-optimal stable mechanism* (DA) of Gale and Shapley (1962) has become one of the most popular mechanisms for distributing school places among students. The relevance of these initiatives and their characteristics are rooted in the theoretical results laid out in the seminal works of Roth (1985), Roth and Sotomayor (1989), Balinski and Sönmez (1999), Abdulkadiroğlu and Sönmez (2003), and Pathak and Sönmez (2013).[2]

The success of DA might be explained by its properties. When students have strict preferences for schools, it is well-known that DA is the only stable and strategy-proof mechanism (Dubins and Freedman, 1981; Roth, 1982; Alcalde and Barberà, 1994). Stability ensures that school places are not wasted and that no one has justified envy towards the assignment of another applicant. Strategy-proofness guarantees that no student has an incentive to misreport her preferences.

Date: June, 2024.

We thank (in alphabetic order) Gabriela Denis, Haluk Ergin, and Federico Echenique for their comments and suggestions. Financial support from ANII, FCE_1_2023_1_176072, is gratefully acknowledged.

E. Duque, School of Business and Economics, Universidad de los Andes, Chile (eduque@uandes.cl).

J. S. Pereyra, Universidad de Montevideo, Uruguay (jspereyra@um.edu.uy).

J. P. Torres-Martínez, Department of Economics, University of Chile (juan.torres@fen.uchile.cl)

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[1] The website www.ccas-project.org provides information about school choice systems in different countries.

[2] We refer to the surveys of Abdulkadiroğlu (2013), Kojima (2017), and Pathak (2017) for a discussion of recent developments in school choice and their applications.

However, when DA is implemented, a student could misrepresent her preferences in order to modify the assignments of others without changing her own school (Roth, 1982). In technical terms, DA is *bossy*. This drawback cannot be avoided unless strong restrictions on the heterogeneity of school priorities are imposed (Ergin, 2002), which compromise the possibility of prioritizing students according to geographical criteria or the presence of siblings in a school.

The bossiness of DA is important for several reasons. First, it is associated with a form of unfairness. One could argue that it is not fair for a student if her assignment is influenced by changes in another student's reported preferences, especially when those changes do not affect the latter's assignment. Second, the bossiness of DA is related to its Pareto inefficiency and to the fact that a coalition of agents could improve their assignments by jointly misrepresenting their preferences (Papái, 2000; Ergin, 2002). Third, it has significant consequences for incentives when students care not only about the school to which they are assigned but also about the assignment of others. In this context, the bossiness of DA compromises the existence of a stable and strategy-proof mechanism, even when each student prioritizes her own school (Duque and Torres-Martínez, 2023).

In this paper, we first introduce a new incentive property satisfied by DA that limits the effects of its bossiness. The new property, called *local non-bossiness*, guarantees that a student cannot change her classmates without changing the school to which she is assigned. Thus, a student's ability to modify others' assignment without changing her school is limited to those who are not assigned to the same school. Second, we use this property to show the existence of a stable and strategy-proof mechanism when students care first about the school to which they are assigned, and then about their classmates. Consequently, the incompatibility between stability and strategy-proofness in contexts where students prioritize their own school is not generated by the existence of preferences over the assignment of others *per se*, but by the fact that these preferences extend beyond their classmates. Evidently, it is reasonable to maintain the dependence of preferences on classmates because it is a well-documented empirical phenomenon in school choice (Rothstein, 2006; Abdulkadiroğlu et al., 2020; Allende, 2021; Che et al., 2022; Beuermann et al., 2023; Cox et al., 2023).

Our analysis begins by showing that DA is locally non-bossy (see Theorem 1) and that this weaker version of non-bossiness is independent of stability or strategy-proofness. Furthermore, any strategy-proof and locally non-bossy mechanism is *locally group strategy-proof*, in the sense that no coalition of classmates can manipulate it to improve the situation for at least one of its members. In particular, DA is the only stable and locally group strategy-proof mechanism (see Corollary 1). We also study the relationships between local non-bossiness and other incentive properties (see details in Appendix A and refer to Figure 1 for a summary of our results).

We then analyze the implications of the local non-bossiness of DA when students care not only about the school to which they are assigned, but also about the assignment of others. It is well-known that many of the results in the literature break down in this case.[3] Nonetheless, one way to recover the existence of a stable matching is by restricting preferences to be *school-lexicographic*. That is, by assuming that each student is primarily concerned with her assigned school, and when assigned to the same school in two different matchings, there is no restriction on how to compare them (Sasaki and Toda, 1996; Dutta and Massó, 1997; Fonseca-Mairena and Triossi, 2023).

However, even in this restricted domain, a stable and strategy-proof mechanism may not exist (Duque and Torres-Martínez, 2023). To avoid this impossibility, we further restrict the preference domain to the family of *school-lexicographic preferences over colleagues*, assuming that students care first about the school and then only about their classmates. That is, each student is indifferent among all the matchings in which she is assigned to the same school with the same classmates. However, there are no restrictions on the order in which she ranks two matchings that assign her to the same school but with different classmates. In this context, we demonstrate that DA induces a stable and strategy-proof mechanism (see Theorem 2). Intuitively, under school-lexicographic preferences over colleagues, a student may want to misreport her preferences to either change her school to a preferred one or maintain her assignment and change her classmates. The first reason for misreporting preferences is already present in classical school choice problems, and avoiding it relates to ensuring strategy-proofness. The second one only emerges in the presence of school-lexicographic preferences, and avoiding it relates to guaranteeing local non-bossiness. Therefore, the strategy-proofness and local non-bossiness of DA are key to ensuring the existence of a stable and strategy-proof mechanism in this context.

Our results have practical implications for admission systems based on the student-optimal stable mechanism. They show that DA still performs well when externalities play a secondary role in students' preferences and focus on the characteristics of their classmates. More formally, despite the variety of mechanisms that could be defined using information on preferences for schools and classmates, applying the DA mechanism to the strict rankings of schools induced by a profile of school-lexicographic preferences defines the only stable and strategy-proof mechanism when students care only about their classmates (see Corollary 2). Furthermore, the impossibility results of Duque and Torres-Martínez (2023) imply that, to some extent, the school-lexicographic preferences over colleagues constitute the maximal domain where a stable and strategy-proof mechanism exists (see Remark 1).

Related literature. We contribute to two strands of the literature: the analysis of the bossiness of DA and the study of incentives in school choice problems with externalities.

The concept of non-bossiness was first introduced by Satterthwaite and Sonnenschein (1981) and has been studied extensively in the context of the assignment of indivisible goods.[4] Non-bossiness plays an essential role in avoiding coalitional manipulability when agents have strict preferences, as Papái (2000) shows that group strategy-proofness

[3]For instance, strong restrictions are required to guarantee the existence of stable matchings, even when students only care about their classmates (Dutta and Massó, 1997; Echenique and Yenmez, 2007; Revilla, 2007; Bodine-Baron et al., 2011; Pycia, 2012; Bykhovskaya, 2020; Pycia and Yenmez, 2023; Cox et al., 2023).

[4]We refer to the critical survey by Thomson (2016) for a detailed discussion of the multiple interpretations of non-bossiness and its implications.

is equivalent to strategy-proofness and non-bossiness. Moreover, Ergin (2002) shows that DA is bossy if and only it is Pareto inefficient. He also characterizes the set of schools' priority orders under which DA is non-bossy. Kojima (2010) demonstrates the incompatibility between non-bossiness and stability in college admission problems, a scenario where both sides of the market may act strategically. However, when only one side of the market reports preferences, as in school choice problems, Afacan and Dur (2017) show that the school-optimal stable mechanism is non-bossy for the students. We contribute to this strand of the literature by highlighting that the bossiness of DA is limited: when it is implemented, no student can change her classmates without also changing her assigned school. This property implies that no group of classmates can manipulate DA to either improve the assignment of at least one of them or maintain the school and change the other colleagues.

In matching problems with externalities, an agent cares not only about her match but also about the distribution of others. This literature was initiated by Sasaki and Toda (1996), who studied stability concepts in marriage markets. In recent years, several authors have extended the analysis to many-to-one matching models and other related allocation problems. They have introduced restrictions in preference domains to specify the type of externalities and ensure the existence of stable outcomes.[5] In the context of many-to-one matching problems, Dutta and Masso (1997) studied a two-sided model with workers and firms where workers' preferences are lexicographic. When workers' preferences over firms dictate their overall preferences over firm-colleague pairs, they show that the set of stable matchings is non-empty (cf., Fonseca-Mairena and Triossi, 2023). Complementing this approach, Echenique and Yenmez (2007) assume that workers have general preferences over colleagues and present an algorithm that produces a set of allocations containing all stable matchings, if they exist. Moreover, Revilla (2007) and Bykhovskaya (2020) determine subdomains of preferences over colleagues where a stable matching always exists.

Kominers (2010) formalizes and proves the idea that the problem with preferences over colleagues is actually more difficult than the classical many-to-one matching problem. Specifically, he shows that every classical many-to-one matching problem may be solved in the setting of Echenique and Yenmez (2007). More recently, Pycia and Yenmez (2023) studied a hybrid model that allows for general types of externalities, including school choice problems as a particular case. They demonstrate that a stable matching exists as long as externalities affect students' choice rules in such a way that a *sustitutability condition* is satisfied, which is also necessary for stability to some extent. Although Dutta and Masso (1997) ensure that school-lexicographic preferences have no effects on stability, Duque and Torres-Martínez (2023) show that a stable and strategy-proof mechanism may not exist. We contribute to this strand of the literature by identifying a new preference domain where this incompatibility does not hold. In particular, when students first consider the school to which they are assigned and then only their classmates, DA induces the unique stable and strategy-proof mechanism. Moreover, it is the local non-bossiness of DA that underlies this result.

[5]The works of Bando, Kawasaki, and Muto (2016) and Pycia and Yenmez (2023) provide excellent surveys on the evolution of this literature.

The rest of the paper is organized as follows. In Section 2 we describe the classical school choice problem and the concepts of local non-bossiness and local group strategy-proofness. In Section 3 we show that DA is locally non-bossy. In Section 4 we introduce a school choice context with externalities, and we show that DA induces a stable and strategy-proof mechanism in the domain of school-lexicographic preferences over colleagues. Section 5 contains the final remarks. Omitted proofs and examples are left to the appendices.

2. Model

Let N be a set of students and S a set of schools. Each $s \in S$ has a priority order $\succ_s$ for students, and a capacity $q_s \geq 1$. Every $i \in N$ is characterized by a complete, transitive, and strict preference relation P_i defined in $S \cup \{s_0\}$, where s_0 represents an outside option. Denote by R_i the weak preference induced by P_i. Given $\succ = (\succ_s)_{s \in S}$, $q = (q_s)_{s \in S}$, and $P = (P_i)_{i \in N}$, we refer to $[N, S, \succ, q]$ as a **school choice context** and to $[N, S, \succ, q, P]$ as a **school choice problem**.

A matching is a function $\mu : N \to S \cup \{s_0\}$ that assigns students to schools such that at most q_s students are assigned to $s \in S$. When $\mu(i) = s_0$, the student i is not assigned to any school. Let $\mathcal{M}$ be the set of matchings and denote by $\mu(s) = \{i \in N : \mu(i) = s\}$ the set of students that μ assigns to $s \in S \cup \{s_0\}$. A matching μ is **individually rational** if no student i prefers s_0 to $\mu(i)$. Moreover, μ is **stable** if it is individually rational and there is no $(i, s) \in N \times S$ such that $sP_i\mu(i)$ and either $|\mu(s)| < q_s$ or $i \succ_s j$ for some $j \in \mu(s)$.

A mechanism is a function that associates a matching to each school choice problem. There is a mechanism designer who knows the school choice context $[N, S, \succ, q]$ and that students' preferences belong to the preference domain $\mathcal{P} = \mathcal{L}^{|N|}$, where $\mathcal{L}$ is the set of strict linear orders defined on $S \cup \{s_0\}$. Given $P = (P_i)_{i \in N} \in \mathcal{P}$ and $C \subseteq N$, let $P_C = (P_i)_{i \in C}$ and $P_{-C} = (P_i)_{i \notin C}$.

When the school choice context is fixed, we will include only the preferences of the students as the argument of a mechanism $\Phi : \mathcal{P} \to \mathcal{M}$, that is $\Phi(P)$ instead of $\Phi(N, S, \succ, q, P)$. Denote by $\Phi_i(P_i, P_{-i})$ the assignment of i when she declares preferences P_i and the other students declare $P_{-i} = (P_j)_{j \neq i}$. In the same way, $\Phi_s(P_i, P_{-i})$ denotes the set of students assigned to $s \in S \cup \{s_0\}$. Consider the following properties:

- Φ is **stable** when $\Phi(P)$ is a stable matching for each $P \in \mathcal{P}$.
- Φ is **strategy-proof** if there is no $i \in N$ such that $\Phi_i(P'_i, P_{-i}) P_i \Phi_i(P)$ for some $P \in \mathcal{P}$ and $P'_i \in \mathcal{L}$.
- Φ is **non-bossy** when $\Phi_i(P) = \Phi_i(P'_i, P_{-i})$ implies that $\Phi(P) = \Phi(P'_i, P_{-i})$, for each $i \in N$, $P \in \mathcal{P}$, and $P'_i \in \mathcal{L}$.
- Φ is **group strategy-proof** if there are no $P \in \mathcal{P}$, $C \subseteq N$, and $P'_C \in \mathcal{L}^{|C|}$ such that:
  - $\Phi_i(P'_C, P_{-C})\, P_i\, \Phi_i(P)$ for some $i \in C$.
  - For each $j \in C$, $\Phi_j(P'_C, P_{-C})\, R_j\, \Phi_j(P)$.

The next is one of the main concept of the paper. Under a bossy mechanism, a student can change the assignment of other students without changing her own. If instead of considering any student, we look only at those who are assigned to the same school as the student who change her preferences, we have a *local* version of non-bossy.

Definition 1. *A mechanism Φ is* ***locally non-bossy*** *when $\Phi_i(P) = \Phi_i(P'_i, P_{-i}) = s$ implies that $\Phi_s(P) = \Phi_s(P'_i, P_{-i})$, for each $i \in N$, $P \in \mathcal{P}$, and $P'_i \in \mathcal{L}$.*

When a mechanism is locally non-bossy, no student can manipulate it to modify her classmates without also changing the school to which she is assigned.

A mechanism is not group strategy-proof if there is a group of students who can benefit by manipulating their preferences. This group may originally be assigned to any school. When we restrict the group of students to be assigned to the same school, we have a *local* version of group strategy-proofness.

Definition 2. *A mechanism Φ is* ***locally group strategy-proof*** *if there are no $s \in S \cup \{s_0\}$, $P \in \mathcal{P}$, $C \subseteq \Phi_s(P)$, and $P'_C \in \mathcal{L}^{|C|}$ such that:*

- *$\Phi_i(P'_C, P_{-C})\, P_i\, \Phi_i(P)$ for some $i \in C$.*
- *For each $j \in C$, $\Phi_j(P'_C, P_{-C})\, R_j\, \Phi_j(P)$.*

When a mechanism is locally group strategy-proof, no coalition of *classmates* can manipulate it to improve the situation of at least one of its members.

In Appendix A we provide a detailed analysis of the relationships between local non-bossiness, local group strategy-proofness, and other incentive properties (see Figure 1 for a summary of our results). Specifically, we demonstrate that local non-bossiness is independent of stability and strategy-proofness, while local group strategy-proofness is weaker than group strategy-proofness and stronger than strategy-proofness. Furthermore, any strategy-proof and locally non-bossy mechanism is locally group strategy-proof, but a locally group strategy-proof mechanism might be locally bossy. This last result contrasts with the well-known equivalence between group strategy-proofness and the combination of strategy-proofness and non-bossiness (Papái, 2000). Additionally, we show that any strategy-proof and locally non-bossy mechanism satisfies a local notion of group non-bossiness (Afacan, 2012).[6]

3. Local non-bossiness of deferred acceptance

Given a school choice context $[N, S, \succ, q]$, let DA : $\mathcal{P} \to \mathcal{M}$ be the **student-optimal stable mechanism**, which associates to each school choice problem the outcome of the deferred acceptance algorithm when students make proposals.

Student-Proposing Deferred Acceptance

Given $[N, S, (\succ_s, q_s)_{s \in S}, (P_i)_{i \in N}]$, implement the following procedure.

Step 1: Each student i proposes to the best alternative in $S \cup \{s_0\}$ according to P_i.
Each school s provisionally accepts the q_s-highest ranked students according to $\succ_s$, among those that have proposed to it.

[6] A notion related to non-bossiness is *consistency* (Ergin, 2002). A mechanism is consistent if after removing some students with their seats, the rest of the students remain in the same school when the mechanism is applied to the reduced market. It is easy to formulate a local notion of consistency and to show that a locally consistent mechanism is locally non-bossy.

Step $k \geq 2$: Each student i who has not been provisionally accepted in step $k-1$ proposes to the best alternative in $S \cup \{s_0\}$ according to P_i, among those to which she has not previously proposed. Each school s provisionally accepts the q_s-highest ranked students according to $\succ_s$, among those that have proposed to it in this step or were provisionally accepted by s in step $k-1$.

The algorithm terminates at the step in which no proposals are made, and provisional acceptances become definitive.

It is well-known that $\mathrm{DA}_i(P)$ is the best alternative in $S \cup \{s_0\}$ that a student i can reach in a stable matching under P (Gale and Shapley, 1962; Roth, 1985). Moreover, the mechanism DA is strategy-proof (Dubins and Freedman, 1981; Roth, 1982). However, a student can manipulate DA to modify the assignment of others without changing her school (Ergin, 2002). Hence, one may wonder if there is a limit on the impact of these manipulations. In particular, would it be possible for a student under DA to change the set of students assigned to her school? As the following result shows, the answer is negative. That is, a student cannot modify her classmates.

Theorem 1. *The student-optimal stable mechanism is locally non-bossy.*

Proof. Given a school choice context $(S, N, \succ, q)$, fix a student $1 \in N$. Consider profiles $(P_1, P_{-1}), (P_1', P_{-1}) \in \mathcal{P}$ such that $\bar{s} \equiv \mathrm{DA}_1(P_1, P_{-1}) = \mathrm{DA}_1(P_1', P_{-1})$. We want to prove that $\mathrm{DA}_{\bar{s}}(P_1, P_{-1}) = \mathrm{DA}_{\bar{s}}(P_1', P_{-1})$. Let $\mu = \mathrm{DA}(P_1, P_{-1})$ and $\mu' = \mathrm{DA}(P_1', P_{-1})$.

When $\bar{s} = s_0$, the result follows from Lemma 3 because no agent without school in μ may have a place in a school in μ' (see Appendix B). When $\bar{s} \neq s_0$, suppose by contradiction that there exists $i_1 \in \mu(\bar{s})$ such that $\mu'(i_1) \neq \bar{s}$. Without loss of generality, assume that $\mu'(i_1) P_{i_1} \bar{s}$ and denote by I the non-empty set of students that prefer μ' to μ.[7]

Let $G = (V, E)$ be the graph with nodes $V = I \cup \{j : \mu(j) = \mu'(i) \text{ for some } i \in I\}$ and directed edges $E = \{[i, j] : i \in I, \mu(j) = \mu'(i)\}$. Hence, the nodes of G are the students who prefer μ' to μ, jointly with those who are assigned in μ to the schools to which students in I are in μ'. Moreover, there is an edge between every node $i \in I$ and all the students who are assigned to $\mu'(i)$ in the matching μ.

Consider the following concepts:

- A *μ-improving cycle* is a tuple of different students $(i_1, \ldots, i_r)$ such that

$$\mu(i_{l+1}) P_{i_l} \mu(i_l), \qquad \forall l \in \{1, \ldots, r\} \text{ [modulo } r\text{]}.$$

- A μ-improving cycle $(i_1, \ldots, i_r)$ is *implementable by μ'* when

$$\mu'(i_l) = \mu(i_{l+1}), \qquad \forall l \in \{1, \ldots, r\} \text{ [modulo } r\text{]}.$$

- A student k *μ-blocks* $[i, j]$ when $\mu(j) P_k \mu(k)$ and $k \succ_{\mu(j)} i$.
- A student k *blocks a μ-improving cycle* $(i_1, \ldots, i_r)$ when she μ-blocks $[i_l, i_{l+1}]$ for some $l \in \{1, \ldots, r\}$ [modulo r].

[7]When $\mu(\bar{s}) \neq \mu'(\bar{s})$, we have that $|\mu(\bar{s})| = |\mu'(\bar{s})| = q_{\bar{s}}$ (see Lemma 3 in Appendix B). Hence, $\mu(\bar{s}) \setminus \mu'(\bar{s})$ and $\mu'(\bar{s}) \setminus \mu(\bar{s})$ are non-empty sets. If we assume that $\bar{s} P_i \mu'(i)$ for all $i \in \mu(\bar{s}) \setminus \mu'(\bar{s})$, then the stability of μ' implies that $j \succ_{\bar{s}} i$ for all $j \in \mu'(\bar{s})$ and $i \in \mu(\bar{s}) \setminus \mu'(\bar{s})$. Hence, the stability of μ ensures that $\mu(j) P_j \bar{s}$ for all $j \in \mu'(\bar{s}) \setminus \mu(\bar{s})$. Therefore, when there is no $i \in \mu(\bar{s}) \setminus \mu'(\bar{s})$ such that $\mu'(i) P_i \bar{s}$, we can maintain our arguments by simply swapping the roles of (P_1, P_{-1}) and (P_1', P_{-1}).

The following properties are satisfied:

(i) Each student $i \in I$ is part of a μ-improving cycle implementable by μ' and contained in the graph G (see Lemma 4 in Appendix B).

(ii) Any student that μ-blocks an edge in E belongs to V.

If $k \in N$ μ-blocks an edge $[i,j]$, then $\mu(j) P_k \mu(k)$ and $k \succ_{\mu(j)} i$. Since $[i,j] \in E$ implies that $\mu(j) = \mu'(i)$, we have that $\mu'(i) P_k \mu(k)$ and $k \succ_{\mu'(i)} i$. Hence, there are two possibilities:

- If $k \neq 1$, the stability of μ' implies that $\mu'(k) R_k \mu'(i)$. Since $\mu'(i) P_k \mu(k)$, the transitivity of P_k ensures that $k \in I$. Hence, $k \in V$.
- If $k = 1$, the previous argument cannot be applied, because the student 1 has different preferences in (P_1, P_{-1}) and (P'_1, P_{-1}). However, the property (i) ensures that i_1 is part of a μ-improving cycle implementable by μ' that is contained in G. Hence, there is $i \in I$ such that $[i, i_1] \in E$. We conclude that $1 \in V$ because there is an edge between i and every student assigned to the school $\mu(i_1) = \bar{s} = \mu(1)$.

(iii) Each node in G has a positive in-degree.

By construction, each node in $V \setminus I$ has a positive in-degree. Moreover, the property (i) ensures that each student $i \in I$ is part of a μ-improving cycle implementable by μ' that is contained in G. Hence, there exists $j \in V$ such that $[j,i] \in E$.

Let $G' = (V, E')$ be the graph obtained from G through the following edge-replacement procedure: given a student $j \in N$, substitute each $[i,j] \in E$ for which the set $I[i,j] \equiv \{k \in N : k \ \mu\text{-blocks } [i,j]\}$ is non-empty by the directed edge $[\bar{k}, j]$, where $\bar{k}$ is the student with the highest priority at school $\mu(j)$ among those in $I[i,j]$. Notice that, when $[\bar{k}, j] \notin E$, it inclusion is feasible because $\bar{k} \in V$ (see property (ii)).

It follows that G' has no μ-improving cycles. Indeed, as *any* student that μ-blocks an edge in E belongs to V,[8] the construction of the new set of edges E' guarantees that a μ-improving cycle in G' cannot be blocked by any student. Hence, if there is such a cycle, then we can implement it to contradicts the fact that μ is the student-optimal stable matching for (P_1, P_{-1}).

However, the property (iii) ensures that all vertices in V have a positive in-degree in G'. Indeed, when E' is constructed from E, if some directed edges of the form $[i,j]$ are deleted, then an edge who points to j is included. As a consequence, G' has a cycle. Since any cycle in G' is a μ-improving cycle, we obtain a contradiction.[9] □

As Theorem 1 demonstrates, when the DA mechanism is implemented, no individual student can manipulate her preferences to change her classmates without changing her school. But what happens when multiple students, *all at the same school*, change their preferences? Can they change their classmates without changing their school? The answer is negative, and not only for DA, but also for any strategy-proof and locally

[8]Since the student 1 may μ-block edges in E, it is crucial to have $1 \in V$. The existence of $i_1 \in \mu(\bar{s})$ such that $\mu'(i_1) \neq \bar{s}$ is only used to prove it (see the property (ii) for details).

[9]If a student k μ-blocks $[i,j] \in E$, then $\mu(j) P_k \mu(k)$. Hence, each $[\bar{k}, j] \in E' \setminus E$ satisfies $\mu(j) P_{\bar{k}} \mu(\bar{k})$. For this reason, any cycle in G' is a μ-improving cycle. However, a μ-improving cycle in G' is not necessarily implemented by μ'.

non-bossy mechanism (see Lemma 2). Next, what happens if the manipulating students belong to different schools? Is it possible for them to change the classmates of one of them without changing their schools? As Example 1 shows, the answer is affirmative.

Example 1. Assume that $N = \{1, 2, 3, 4\}$, $S = \{s_1, s_2, s_3\}$, $(q_{s_1}, q_{s_2}, q_{s_3}) = (1, 2, 1)$, and

$$4 \succ_{s_1} 2 \succ_{s_1} 3 \succ_{s_1} 1, \qquad 2 \succ_{s_2} 3 \succ_{s_2} 1 \succ_{s_2} 4, \qquad 1 \succ_{s_3} 2 \succ_{s_3} 3 \succ_{s_3} 4.$$

If the preferences of students are such that:

$$s_2 P_1 s_3 P_1 s_1 P_1 s_0, \qquad s_1 P_2 s_2 P_2 s_3 P_2 s_0, \qquad s_1 P_3 s_2 P_3 s_3 P_3 s_0, \qquad s_2 P_4 s_1 P_4 s_3 P_4 s_0,$$

then $\text{DA}(P) = ((1, s_3), (2, s_2), (3, s_2), (4, s_1))$. If we consider preferences (P'_1, P'_2) such that s_3 is the best alternative under P'_1 and s_2 is the best alternative under P'_2, we have that $\text{DA}(P'_1, P'_2, P_3, P_4) = ((1, s_3), (2, s_2), (3, s_1), (4, s_2))$. Therefore, 1 and 2 change their preferences, both remain at the same school, and 2 changes her classmate. □

Notice that, DA is not the only stable and locally non-bossy mechanism that can be defined in $\mathcal{P}$. Indeed, the *school-optimal* stable mechanism also satisfies these properties, because it is non-bossy (Afacan and Dur, 2017).[10] However, since DA is the only mechanism defined in $\mathcal{P}$ that is stable and strategy-proof (Alcalde and Barberà, 1994), the following result is a direct consequence of Theorem 1 and Lemma 1.

Corollary 1. *The* DA *mechanism is the only one that is stable and locally group strategy-proof.*

4. Mechanisms under preferences over colleagues

Most of the literature in school choice assumes that students only care about the school to which they are assigned. However, it is unrealistic that students do not care about the assignment of others. In particular, it has been empirically documented that the composition of classmates is an important consideration when students are choosing a school (Rothstein, 2006; Abdulkadiroğlu et al., 2020; Allende, 2021; Che et al., 2022; Beuermann et al., 2023; Cox et al., 2023).

However, if students have general preferences over the set of matchings many of the results in the literature break down. Specifically, a stable matching may not exist, *even when students only care who the other students assigned to her school are* (Echenique and Yenmez, 2007). One way to overcome this problem is to restrict the domain of preferences. For instance, assuming that students have *school-lexicographic preferences*, meaning that they are primarily concerned with the school assigned (Sasaki and Toda, 1996; Dutta and Massó, 1997).

Under school-lexicographic preferences, some of the standard results are recovered. In particular, the existence of a stable matching. However, there may not exist a stable and strategy proof mechanism (Duque and Torres-Martínez, 2023). Thus, one may wonder if

[10] The arguments made in the proof of Theorem 1 can be easily adapted to provide a direct proof that the school-optimal stable mechanism is locally non-bossy. Indeed, since this mechanism assigns each student to the worst school she can reach in a stable matching, it is sufficient to repeat the proof focusing on *μ-worsening cycles* (i.e., tuples $(i_1, \ldots, i_r)$ such that $(i_r, \ldots, i_1)$ forms a μ-improving cycle).

there is a subdomain of school-lexicographic preferences in which such a mechanism exists. In this section, we show that within the subdomain of school-lexicographic preferences where only the school and classmates matter, the local non-bossiness of DA implies that it induces a stable and strategy-proof mechanism.

We first define the domain of school-lexicographic preferences, and then the subdomain where there exists a stable and strategy-proof mechanism.

Definition 3. *Given a school choice context $[N, S, \succ, q]$, a* ***school-lexicographic preference*** *for student $i \in N$ is a complete and transitive preference relation $\trianglerighteq_i$ defined on $\mathcal{M}$ that satisfies:*

- *If $\mu(i) \neq \eta(i)$, then either $\mu \triangleright_i \eta$ or $\eta \triangleright_i \mu$, where $\triangleright_i$ is the strict part of $\trianglerighteq_i$.*
- *If $\mu \triangleright_i \eta$ and $\mu(i) \neq \eta(i)$, then $\mu' \triangleright_i \eta'$ for all matchings $\mu', \eta' \in \mathcal{M}$ such that $\mu'(i) = \mu(i)$ and $\eta'(i) = \eta(i)$.*

Let $\mathcal{D}$ be the set of preference profiles $\trianglerighteq = (\trianglerighteq_i)_{i \in N}$ satisfying these properties.

The first condition states that a student cannot be indifferent between two matchings where she is assigned to different schools. By the second condition, if a student prefers a matching μ over η, and she is assigned to different schools in these two matchings, then she should prefer every matching where she is assigned to the same school as in μ to every matching where she is assigned to the same school as in η. As we will see below, this last condition allows us to define a preference over schools from $\trianglerighteq_i$.

Notice that, in the preference domain $\mathcal{D}$ no restrictions are imposed over the ranking of two matchings in which a student has the same school. Let $P(\trianglerighteq) = (P_i(\trianglerighteq))_{i \in N} \in \mathcal{P}$ be the preferences over schools induced by $\trianglerighteq = (\trianglerighteq_i)_{i \in N} \in \mathcal{D}$ through the rule $s\, P_i(\trianglerighteq)\, s'$ if and only if there exist $\mu, \mu' \in \mathcal{M}$ such that $\mu(i) = s$, $\mu'(i) = s'$, and $\mu \triangleright_i \mu'$.

One can naturally extend the concepts of stability and strategy-proofness to the domain of school-lexicographic preferences. Given $[N, S, \succ, q, \trianglerighteq]$, where $\trianglerighteq \in \mathcal{D}$, a matching μ is **stable** when for each student i the following conditions are satisfied:

- *No justified envy*: there is no $s \in S$ and $j \in \mu(s)$ such that $sP_i(\trianglerighteq)\mu(i)$ and $i \succ_s j$.
- *Non-wastefulness*: there is no $s \in S$ such that $|\mu(s)| < q_s$ and $sP_i(\trianglerighteq)\mu(i)$.
- *Individual rationality*: $\mu(i)P_i(\trianglerighteq)s_0$ or $\mu(i) = s_0$.

Given $\mathcal{D}' \subseteq \mathcal{D}$, a **stable mechanism** associates to each preference profile $\trianglerighteq \in \mathcal{D}'$ a stable matching of $[N, S, \succ, q, \trianglerighteq]$. A mechanism Γ is **strategy-proof** in $\mathcal{D}'$ when there is no student i such that $\Gamma_i(\trianglerighteq_i', \trianglerighteq_{-i}) \triangleright_i \Gamma_i(\trianglerighteq)$ for some $\trianglerighteq$ and $(\trianglerighteq_i', \trianglerighteq_{-i})$ in $\mathcal{D}'$.

School-lexicographic preferences have no effects on stability, because the school choice problems $[N, S, \succ, q, \trianglerighteq]$ and $[N, S, \succ, q, P(\trianglerighteq)]$ have the same stable matchings (Sasaki and Toda, 1996; Dutta and Massó, 1997; Fonseca-Mairena and Triossi, 2023). However, there are strong effects on incentives, as no stable mechanism is strategy-proof (Duque and Torres-Martínez, 2023).

Within the domain of school-lexicographic preferences, let us consider a scenario where students are solely concerned about the composition of the school where they are assigned. Consequently, a student will be indifferent between two matchings where she is assigned to the same school with the same classmates. This is the idea of the following subdomain of preferences.

Definition 4. *Given $[N, S, \succ, q]$, the domain of* ***preferences over colleagues*** $\mathcal{D}^{\mathrm{p}} \subseteq \mathcal{D}$ *is the set of profiles* $(\trianglerighteq_i)_{i \in N}$ *such that* $\mu \rhd_i \eta$ *and* $\mu(i) = \eta(i) = s$ *imply that* $\mu(s) \neq \eta(s)$.

In words, when $(\trianglerighteq_i)_{i \in N}$ belongs to $\mathcal{D}^{\mathrm{p}}$ each student i strictly prefers μ to η only when she has different schools or different classmates in these matchings.

The next result shows that there is a stable and strategy-proof mechanism in $\mathcal{D}^{\mathrm{p}}$, which is the student optimal stable mechanism applied to $[N, S, \succ, q, P(\trianglerighteq)]$.

Theorem 2. *In any school choice context* $(S, N, \succ, q)$, *the mechanism* $\overline{\mathrm{DA}} : \mathcal{D}^{\mathrm{p}} \to \mathcal{M}$ *defined by* $\overline{\mathrm{DA}}(\trianglerighteq) = \mathrm{DA}(P(\trianglerighteq))$ *is stable and strategy-proof.*

Proof. Fix $\trianglerighteq \in \mathcal{D}^{\mathrm{p}}$. The stability of $\overline{\mathrm{DA}}(\trianglerighteq)$ is a consequence of the fact that $\mathrm{DA} : \mathcal{P} \to \mathcal{M}$ is a stable mechanism, because $[N, S, \succ, q, \trianglerighteq]$ and $[N, S, \succ, q, P(\trianglerighteq)]$ has the same stable matchings. By contradiction, assume that $\overline{\mathrm{DA}}$ is not strategy-proof in $\trianglerighteq$. Hence, there exists $i \in N$ such that $\overline{\mathrm{DA}}(\trianglerighteq_i', \trianglerighteq_{-i}) \rhd_i \overline{\mathrm{DA}}(\trianglerighteq)$ for some $\trianglerighteq_i'$ such that $\trianglerighteq' \equiv (\trianglerighteq_i', \trianglerighteq_{-i})$ belongs to $\mathcal{D}^{\mathrm{p}}$. Since $\overline{\mathrm{DA}}(\trianglerighteq_i', \trianglerighteq_{-i}) \neq \overline{\mathrm{DA}}(\trianglerighteq)$, we have two relevant cases:

- When $\overline{\mathrm{DA}}_i(\trianglerighteq_i', \trianglerighteq_{-i}) \neq \overline{\mathrm{DA}}_i(\trianglerighteq)$, the definitions of $\overline{\mathrm{DA}}$ and $P_i(\trianglerighteq)$ ensure that
$$\mathrm{DA}_i(P_i(\trianglerighteq'), P_{-i}(\trianglerighteq)) \; P_i(\trianglerighteq) \; \mathrm{DA}_i(P(\trianglerighteq)).$$
This implies that DA is not strategy-proof in $\mathcal{P}$. A contradiction.
- When $\overline{\mathrm{DA}}_i(\trianglerighteq_i', \trianglerighteq_{-i}) = \overline{\mathrm{DA}}_i(\trianglerighteq) = s$, the definitions of $\overline{\mathrm{DA}}$ and $\mathcal{D}^{\mathrm{p}}$ ensure that
$$\begin{aligned} \mathrm{DA}_i(P_i(\trianglerighteq'), P_{-i}(\trianglerighteq)) &= \mathrm{DA}_i(P(\trianglerighteq)) = s, \\ \mathrm{DA}_s(P_i(\trianglerighteq'), P_{-i}(\trianglerighteq)) &\neq \mathrm{DA}_s(P(\trianglerighteq)). \end{aligned}$$
This implies that DA is locally bossy. A contradiction (see Theorem 1).

Therefore, the mechanism $\overline{\mathrm{DA}} : \mathcal{D}^{\mathrm{p}} \to \mathcal{M}$ is strategy-proof. □

Remark 1. *To some extent, Theorem 2 is no longer valid beyond the domain* $\mathcal{D}^{\mathrm{p}}$. *Indeed, when* $(\succ, q)$ *has Ergin-cycles, there is* $\mathcal{D}' \subseteq \mathcal{D}$ *such that no stable mechanism defined in* $\mathcal{D}'$ *is strategy-proof (Duque and Torres-Martínez, 2023).*[11] *Moreover, for some specifications of* $(\succ, q)$, *only one student is required to have unrestricted school-lexicographic preferences in* $\mathcal{D}'$, *while the others may have preferences over colleagues.*[12] □

A stable mechanism $\Phi : \mathcal{P} \to \mathcal{M}$ always induces a stable mechanism defined in $\mathcal{D}^{\mathrm{p}}$ by the rule that associates each $\trianglerighteq \in \mathcal{D}^{\mathrm{p}}$ with the matching $\Phi(P(\trianglerighteq))$. Evidently, many other stable mechanisms can be defined in $\mathcal{D}^{\mathrm{p}}$ using the information that students reveal about how they rank their colleagues. Despite this multiplicity, the uniqueness result of Alcalde and Barberà (1994) can be extended to the preference domain $\mathcal{D}^{\mathrm{p}}$.

[11] The pair $(\succ, q)$ has an Ergin-cycle when there are $s, s' \in S$ and $i, j, k \in N$ such that $i \succ_s j \succ_s k \succ_{s'} i$ and a scarcity condition that causes students $\{i, j, k\}$ to compete for schools $\{s, s'\}$ is satisfied (see Ergin, 2002).

[12] For details, see the proof of Theorem 1 in Duque and Torres-Martínez (2023).

Corollary 2. $\overline{\text{DA}}$ *is the only mechanism that is stable and strategy-proof in the domain* $\mathcal{D}^{\text{p}}$.

Proof. Theorem 2 ensures that the mechanism $\overline{\text{DA}}$ is stable and strategy-proof. By contradiction, assume that there is a stable and strategy-proof mechanism $\Omega : \mathcal{D}^{\text{p}} \to \mathcal{M}$ such that $\Omega(\trianglerighteq) \neq \overline{\text{DA}}(\trianglerighteq)$ for some $\trianglerighteq \in \mathcal{D}^{\text{p}}$. Since $\mathcal{D}^{\text{p}}$ only includes school-lexicographic preferences, $\overline{\text{DA}}(\trianglerighteq)$ pairs each student to the best alternative in $S \cup \{s_0\}$ that she can obtain in a stable outcome of $(S, N, \succ, q, \trianglerighteq)$.[13] Thus, we have that $\overline{\text{DA}}_i(\trianglerighteq) \rhd_i \Omega_i(\trianglerighteq)$ for some $i \in N$. In particular, $\overline{\text{DA}}_i(\trianglerighteq)$ is a school.

Let P'_i be the preference defined on $S \cup \{s_0\}$ that only considers $\overline{\text{DA}}_i(\trianglerighteq)$ acceptable (i.e., $s_0 P'_i s$ for all school $s \neq \overline{\text{DA}}_i(\trianglerighteq)$). Fix $\trianglerighteq' = (\trianglerighteq'_i, \trianglerighteq_{-i}) \in \mathcal{D}^{\text{p}}$ such that $P_i(\trianglerighteq') = P'_i$. Since the problems $(S, N, \succ, q, \trianglerighteq')$ and $(S, N, \succ, q, (P'_i, P_{-i}(\trianglerighteq)))$ have the same stable matchings, and $\overline{\text{DA}}(\trianglerighteq)$ is stable under $(P'_i, P_{-i}(\trianglerighteq))$, the definition of P'_i and the Rural Hospital Theorem (Roth, 1986) imply that $\Omega_i(\trianglerighteq'_i, \trianglerighteq_{-i}) = \overline{\text{DA}}_i(\trianglerighteq)$. Therefore, $\Omega_i(\trianglerighteq'_i, \trianglerighteq_{-i}) \rhd_i \Omega_i(\trianglerighteq)$, which contradicts the strategy-proofness of Ω. □

5. Concluding remarks

In this paper, we have shown that one of the main drawbacks of the student-optimal stable mechanism, its bossiness, is limited. When DA is implemented, it is well-known that a student can modify another student's assignment by changing her preferences while remaining in the same school. We have demonstrated that this holds only for students not assigned to the same school. In other words, under DA, a student cannot change her classmates without changing her own school. Additionally, for any strategy-proof mechanism, local non-bossiness guarantees that no coalition of students assigned to the same school can misrepresent their preferences to either improve their assignments or maintain their school while modifying their colleagues. Since DA is not group strategy-proof, our result implies that a successful manipulating coalition of DA must include students from different schools.

Local non-bossiness is not only interesting in itself but also because of its application to school choice problems with externalities. In this framework, even when students prioritize their assigned school first and then consider the assignment of others, a stable and strategy-proof mechanism may not exist (Duque and Torres-Martínez, 2023). However, if we restrict preferences to be such that only the school and the classmates matter, it turns out that DA induces the only stable and strategy-proof mechanism. The crucial property behind this result is precisely its local non-bossiness.

Although we have focused on many-to-one matching problems with responsive preferences, it might be interesting to investigate the role of local non-bossiness in more general frameworks, as many-to-one matching models with contracts (Hatfield and Milgrom, 2005) or many-to-many matching problems (Echenique and Oviedo, 2006).

These extensions are of interest due to their potential applications to more complex labor or educational markets, such as school choice with affirmative action or part-time work assignments (e.g., medical interns in hospitals or teachers in public schools). However, to guarantee that a mechanism Φ is locally non-bossy using our approach, the following three properties are crucial: (i) Φ must be a stable mechanism; (ii) Φ must

[13]That is, there is no stable matching μ such that $\mu(i) P_i(\trianglerighteq) \overline{\text{DA}}_i(\trianglerighteq)$ for some $i \in N$.

assign all students to the best/worst school they can attend in a stable matching; and (iii) any school that does not fill its quota in some stable matching must enroll the same set of students in any other stable outcome. Therefore, any attempt to extend our results to other contexts will likely require restricting the preference domains to ensure that properties analogous to (i)-(iii) hold. This is a matter for future research.

Appendix A. Local non-bosiness and its implications

Lemma 1. *Consider a mechanism $\Phi : \mathcal{P} \to \mathcal{M}$. If Φ is strategy-proof and locally non-bossy, then it is locally group strategy-proof.*

Proof. Assume that the mechanism Φ is strategy-proof and locally non-bossy. Given $P \in \mathcal{P}$ and $s \in S \cup \{s_0\}$, suppose that there is a coalition $C = \{i_1, \dots, i_r\}$ contained in $\Phi_s(P)$ such that $\Phi_i(P'_C, P_{-C}) R_i \Phi_i(P)$ for all $i \in C$ and for some $P'_C = (P'_j)_{j \in C}$. We want to prove that $\Phi_i(P'_C, P_{-C}) = \Phi_i(P)$ for all $i \in C$.

For each $i \in \Phi_s(P)$, let $\widehat{P}_i$ be the preference relation that puts $\Phi_i(P'_C, P_{-C})$ at the top and keeps the other schools in the order induced by P_i. Suppose that $\Phi_{i_1}(P) \neq \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1})$. The strategy-proofness of Φ implies that $\Phi_{i_1}(P) P_{i_1} \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1})$ and $\Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) \widehat{P}_{i_1} \Phi_{i_1}(P)$. If $\Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) \neq \Phi_{i_1}(P'_C, P_{-C})$, the definition of $\widehat{P}_{i_1}$ implies that $\Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) P_i \Phi_{i_1}(P)$. Hence, $\Phi_{i_1}(P) P_{i_1} \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) P_{i_1} \Phi_{i_1}(P)$, which does not make sense. As a consequence, $\Phi_{i_1}(P) \neq \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1})$ ensures that $\Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1}) = \Phi_{i_1}(P'_C, P_{-C})$, which in turn implies that $\Phi_{i_1}(P) P_{i_1} \Phi_{i_1}(P'_C, P_{-C})$, a contradiction. We conclude that $\Phi_{i_1}(P) = \Phi_{i_1}(\widehat{P}_{i_1}, P_{-i_1})$, and the local non-bossiness of Φ implies that $\Phi_i(P) = \Phi_i(\widehat{P}_{i_1}, P_{-i_1})$ for all$i \in \Phi_s(P)$. Repeating the argument for students $i_2, \dots, i_r$ we obtain that $\Phi_i(P) = \Phi_i(\widehat{P}_C, P_{-C})$ for all $i \in \Phi_s(P)$.

As $\Phi_{i_1}(P'_C, P_{-C})$ is the best alternative under $\widehat{P}_{i_1}$, $\Phi_{i_1}(P'_C, P_{-C}) \widehat{R}_{i_1} \Phi_{i_1}(\widehat{P}_{i_1}, P'_{C\setminus\{i_1\}}, P_{-C})$. If $\Phi_{i_1}(P'_C, P_{-C}) \neq \Phi_{i_1}(\widehat{P}_{i_1}, P'_{C\setminus\{i_1\}}, P_{-C})$, then $\Phi_{i_1}(P'_C, P_{-C}) \widehat{P}_{i_1} \Phi_{i_1}(\widehat{P}_{i_1}, P'_{C\setminus\{i_1\}}, P_{-C})$, which contradicts the strategy-proofness of Φ. Moreover, as Φ is locally non-bossy, we have that $\Phi_i(P'_C, P_{-C}) = \Phi_i(\widehat{P}_{i_1}, P'_{C\setminus\{i_1\}}, P_{-C})$ for all$i \in \Phi_s(P)$. Repeating the same argument for $i_2, \dots, i_r$, we conclude that $\Phi_i(P'_C, P_{-C}) = \Phi_i(\widehat{P}_C, P_{-C})$ for all $i \in \Phi_s(P)$.

In particular, we have that $\Phi_i(P'_C, P_{-C}) = \Phi_i(P)$ for every student $i \in C$. □

In Example 3 we show that the converse of Lemma 1 does not hold.

Afacan (2012) introduces the following extension of non-bossiness to group of agents: a mechanism Φ is **group non-bossy** when for any $P \in \mathcal{P}$ and $C \subseteq N$, if there exists $P'_C \in \mathcal{L}^{|C|}$ such that $\Phi_i(P) = \Phi_i(P'_C, P_{-C})$ for all $i \in C$, then $\Phi(P) = \Phi(P'_C, P_{-C})$.

We will consider a local version of this property: a mechanism Φ is **locally group non-bossy** when for any $s \in S \cup \{s_0\}$, $P \in \mathcal{P}$, and $C \subseteq \Phi_s(P)$, if there exists $P'_C \in \mathcal{L}^{|C|}$ such that $\Phi_i(P) = \Phi_i(P'_C, P_{-C})$ for all $i \in C$, then $\Phi_s(P) = \Phi_s(P'_C, P_{-C})$.

Hence, local group non-bossiness ensures that no coalition of students assigned to the same school can misreport their preferences to change some classmates without modifying their school.

Lemma 2. *Consider a mechanism $\Phi : \mathcal{P} \to \mathcal{M}$. If Φ is strategy-proof and locally non-bossy, then it is locally group non-bossy.*

Proof. Assume that the mechanism Φ is strategy-proof and locally non-bossy. Given $s \in S \cup \{s_0\}$, $P \in \mathcal{P}$, $C \subseteq \Phi_s(P)$, and $P'_C \in \mathcal{L}^{|C|}$, suppose that $\Phi_i(P) = \Phi_i(P'_C, P_{-C})$ for all $i \in C$. Since $\Phi_i(P) = \Phi_i(P'_C, P_{-C})$ ensures that $\Phi_i(P'_C, P_{-C}) R_i \Phi_i(P)$, the arguments made in the proof of Lemma 1 imply that $\Phi_j(P) = \Phi_j(P'_C, P_{-C})$ for all $j \in \Phi_s(P)$. Hence, $\Phi_s(P) \subseteq \Phi_s(P'_C, P_{-C})$. Moreover, swapping the roles of P and (P'_C, P_{-C}), we can ensure that $\Phi_s(P'_C, P_{-C}) \subseteq \Phi_s(P)$. Therefore, $\Phi_s(P) = \Phi_s(P'_C, P_{-C})$. □

The converse of Lemma 2 does not hold. Indeed, consider the *Boston mechanism*, also known as the Immediate Acceptance mechanism. This mechanism runs similarly to DA, with the difference that at each step accepted students are definitively matched to the school. Although it is not strategy-proof (Abdulkadiroğlu and Sönmez, 2003), it is locally group non-bossy. Formally, denoting by $\mathcal{B} : \mathcal{P} \to \mathcal{M}$ this mechanism, suppose that $\mathcal{B}_i(P) = \mathcal{B}_i(P'_C, P_{-C}) = s$ for all student in a coalition $C \subseteq \mathcal{B}_s(P)$. Let $\widehat{P}_C = (\widehat{P}_i)_{i \in C}$ be such that s is the best alternative under each $\widehat{P}_i$. It is not difficult to verify that $\mathcal{B}_s(P) = \mathcal{B}_s(\widehat{P}_C, P_{-C}) = \mathcal{B}_s(P'_C, P_{-C})$.

In what follows, we present examples that allow us to determine the relationships between local non-bossiness and other properties. In particular, we will conclude that local non-bossiness is independent of stability (see Example 2) and strategy-proofness (see Example 3).

Example 2. (*A stable mechanism that is locally bossy*)
Suppose that $N = \{1, 2, 3\}$, $S = \{s_1, s_2\}$, $(q_{s_1}, q_{s_2}) = (2, 1)$,

$$1 \succ_{s_1} 2 \succ_{s_1} 3, \qquad \text{and} \qquad 3 \succ_{s_2} 2 \succ_{s_2} 1.$$

Fix a preference profile $\overline{P} = (\overline{P}_1, \overline{P}_2, \overline{P}_3)$ such that:

$$s_1 \overline{P}_1 s_2 \overline{P}_1 s_0, \qquad s_2 \overline{P}_2 s_1 \overline{P}_2 s_0, \qquad s_1 \overline{P}_3 s_2 \overline{P}_3 s_0.$$

Since the *school-optimal* stable mechanism in $[N, S, q, \succ, \overline{P}]$ is

$$\mathrm{DA}^S(\overline{P}) = ((1, s_1), (2, s_1), (3, s_2)),$$

it differs from $\mathrm{DA}(\overline{P}) = ((1, s_1), (2, s_2), (3, s_1))$. Let $\Omega : \mathcal{P} \to \mathcal{M}$ be the stable mechanism such that $\Omega(P) = \mathrm{DA}(P)$ when $P \neq \overline{P}$, and $\Omega(\overline{P}) = \mathrm{DA}^S(\overline{P})$.

We claim that Ω is locally bossy. Let P_1 be such that $s_2 P_1 s_1 P_1 s_0$. It is easy to see that $\Omega(P_1, \overline{P}_{-1}) = \mathrm{DA}(P_1, \overline{P}_{-1}) = ((1, s_1), (2, s_2), (3, s_1))$. Since $\Omega(\overline{P}_1, \overline{P}_{-1}) = \mathrm{DA}^S(\overline{P})$, when the student 1 changes her preferences from P_1 to $\overline{P}_1$, she remains assigned to school s_1, but her classmate changes as $\Omega_{s_1}(\overline{P}_1, \overline{P}_{-1}) = \{1, 2\} \neq \{1, 3\} = \Omega_{s_1}(P_1, \overline{P}_{-1})$. □

Although group strategy-proofness is stronger than group non-bossiness (Afacan, 2012), the following example shows that the analogous result does not hold for the local versions of these concepts.

Example 3. (*A locally group strategy-proof mechanism that is locally bossy*)

Suppose that $N = \{1,2,3\}$, $S = \{s_1, s_2\}$, $(q_{s_1}, q_{s_2}) = (2,1)$,

$$1 \succ_{s_1} 2 \succ_{s_1} 3, \qquad \text{and} \qquad 3 \succ_{s_2} 1 \succ_{s_2} 2.$$

Let $\Phi : \mathcal{P} \to \mathcal{M}$ be a mechanism such that $\Phi(P) = \text{DA}(P)$ unless the most preferred school of student 1 is s_1. In this later case, let $\Phi_1(P) = s_1$ and $\Phi_i(P) = s_0$ for all $i \neq 1$. Notice that, since 1 has the highest priority at s_1, $\Phi_1(P) R_1 s_1$ for all $P \in \mathcal{P}$.

The mechanism Φ is strategy-proof, because DA satisfies this property and no one can prevent 1 to receive a seat in s_1 when it is her best alternative. Moreover, Φ is locally group strategy-proof. Indeed, if this is not the case, there exist $P, P' \in \mathcal{P}$ and two agents in $\Phi_{s_1}(P) = \{i, j\}$ such that if they report (P'_i, P'_j) none of them are worse off, and at least one of them is strictly better off. Note that s_1 is not the most preferred school for student 1 under either P_1 (because in this case only one student is assigned to s_1) or P'_1 (because otherwise at least one of the agents $\{i, j\}$ is not assigned under (P'_i, P'_j, P_k) and her situation worsens). Thus, Φ coincides with DA in P and (P'_i, P'_j, P_k). This contradicts the local group strategy-proofness of DA (see Theorem 1 and Lemma 1).

However, Φ is locally bossy. Consider the preference profile $P = (P_1, P_2, P_3)$:

$$s_2 P_1 s_1 P_1 s_0, \qquad s_1 P_2 s_2 P_2 s_0, \qquad s_2 P_3 s_1 P_3 s_0.$$

We have that $\Phi(P) = ((1, s_1), (2, s_1), (3, s_2))$ and $\Phi_{s_1}(P) = \{1, 2\}$. If the student 1 changes her preference to P'_1 such that $s_1 P'_1 s_2 P'_1 s_0$, she remains assigned to school s_1 but is left without classmates because $\Phi_{s_1}(P'_1, P_2, P_3) = \{1\}$. □

Since DA is stable and strategy-proof in $\mathcal{P}$ (Dubins and Freedman, 1981; Roth, 1982), examples 2 and 3 guarantee that any pair of properties between stability, strategy-proofness, and local bossiness are compatible with each other. However, no mechanism satisfies the three properties, because DA is the only stable and strategy-proof mechanism (Alcalde and Barberà, 1994) and our Theorem 1 shows that it is locally non-bossy.

The last examples show that local group non-bossiness is stronger than local non-bossiness (see Example 4), and than local group strategy-proofness is stronger that strategy-proofness (see Example 5).

Example 4. (*A locally non-bossy mechanism that is not locally group non-bossy*)

Suppose that $N = \{1,2,3\}$, $S = \{s_1, s_2\}$, $(q_{s_1}, q_{s_2}) = (3,1)$,

$$1 \succ_{s_1} 2 \succ_{s_1} 3, \qquad \text{and} \qquad 3 \succ_{s_2} 2 \succ_{s_2} 1.$$

Denote by $top(P_i)$ the best alternative of student i when her preferences are $P_i \in \mathcal{L}$. Let $\Omega : \mathcal{P} \to \mathcal{M}$ be the mechanism such that:

$$\Omega(P) = \begin{cases} ((1, s_1), (2, s_1), (3, s_2)), & \text{when } top(P_1) = top(P_2) = s_1; \\ ((1, s_1), (2, s_1), (3, s_1)), & \text{when } top(P_1) = top(P_2) = s_2; \\ ((1, s_0), (2, s_0), (3, s_0)), & \text{in another case.} \end{cases}$$

It is not difficult to verify that Ω is locally non-bossy: the only two students who might change their partners without changing schools are 1 and 2. However, if only one of them changes her preferences, she will be assigned to s_0 (if she was originally assigned to s_1 or s_2), or will not change anyone's assigned school otherwise.

Moreover, Ω is locally group bossy. Let $P = (P_1, P_2, P_3)$ be a preference profile such that $top(P_1) = top(P_2) = s_1$ and consider $P_1', P_2' \in \mathcal{L}$ such that $top(P_1') = top(P_2') = s_2$. Notice that, although $\Omega_i(P) = \Omega_i(P_1', P_2', P_3) = s_1$ for each student $i \in \{1,2\}$, we have that $\Omega_{s_1}(P) = \{1,2\} \neq \{1,2,3\} = \Omega_{s_1}(P_1', P_2', P_3)$. □

Example 5. (*A strategy-proof mechanism that is not locally group strategy-proof*)
Suppose that $N = \{1,2,3\}$, $S = \{s_1, s_2\}$, $(q_{s_1}, q_{s_2}) = (2,2)$,

$$1 \succ_{s_1} 2 \succ_{s_1} 3, \qquad \text{and} \qquad 3 \succ_{s_2} 2 \succ_{s_2} 1.$$

Denote by $top_k(P_i)$ the k-th best alternative of student i under $P_i \in \mathcal{L}$. Let $\Omega : \mathcal{P} \to \mathcal{M}$ be the mechanism such that:

$$\Omega(P) = \begin{cases} ((1, top_1(P_1)), (2, s_1), (3, s_2)), & \text{when } top_2(P_1) = s_2; \\ ((1, top_1(P_1)), (2, s_2), (3, s_1)), & \text{in another case.} \end{cases}$$

It is easy to verify that Ω is strategy-proof. We claim that Ω is not locally group strategy-proof. Let $P = (P_1, P_2, P_3)$ be a preference profile such that $s_1 P_1 s_2 P_1 s_0$ and $s_2 P_2 s_1 P_2 s_0$. Consider $P_1', P_2' \in \mathcal{L}$ such that $s_1 P_1' s_0 P_1' s_2$ and $P_2' = P_2$. It follows that the coalition of classmates $\{1,2\} \in \Omega_{s_1}(P)$ can manipulate the mechanism Ω at P, because $\Omega_1(P) = s_1 = \Omega_1(P_1', P_2', P_3)$ and $\Omega_2(P_1', P_2', P_3) = s_2 P_2 s_1 = \Omega_2(P)$. □

The following diagram summarizes the causal relationships between local non-bossiness and other incentive properties:

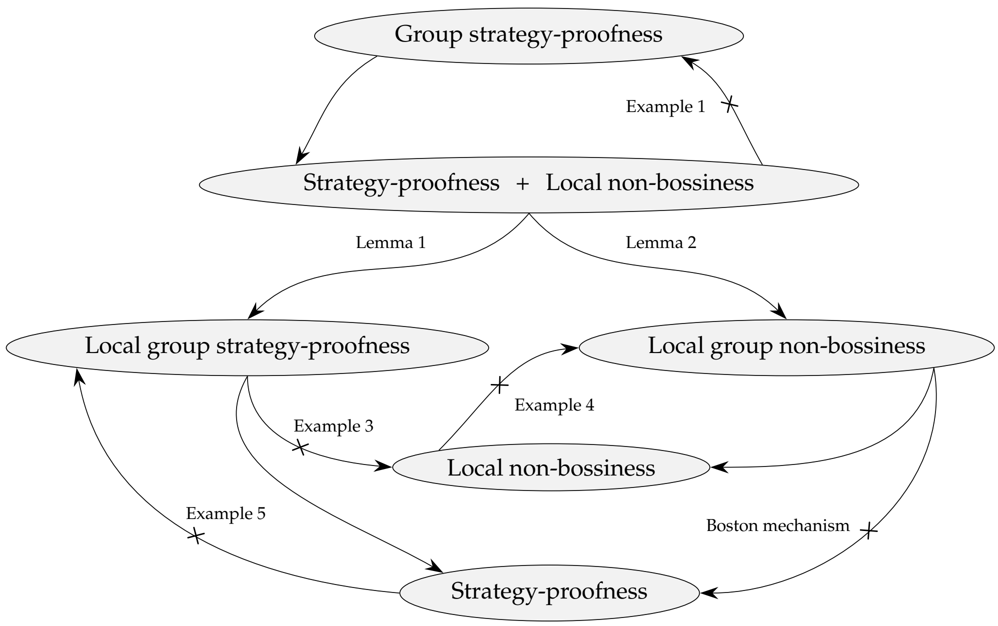

FIGURE 1. On local non-bossiness and other incentive properties.

Notice that, with the information provided in Figure 1, it can be easily inferred that all causal relationships that have not been drawn do not hold.

Appendix B. omitted proofs

Given a student $1 \in N$ and preference profiles $P, P' \in \mathcal{P}$, let $\mu = \mathrm{DA}(P_1, P_{-1})$ and $\mu' = \mathrm{DA}(P_1', P_{-1})$. In the following results we characterize some properties of μ and μ'.

Lemma 3. *Suppose that $\mu(1) = \mu'(1) = \bar{s}$. If there exists $i \in N$ such that $a = \mu(i)$ and $a' = \mu'(i)$ are different, then they are schools and fill their places in μ and μ'.*

Proof. Let $\widehat{\mu}$ be the matching induced by μ when student 1 and one seat in $\bar{s}$ are eliminated (w.l.o.g., assume that $q_{s_0} = |N|$). Define $\widehat{\mu}'$ analogously. Notice that $\widehat{\mu}$ and $\widehat{\mu}'$ are stable matchings in the school choice problem $(N \setminus \{1\}, S, (\succ_s)_{s\in S}, (\widehat{q}_s)_{s\in S}, P_{-1})$ where $\widehat{q}_{\bar{s}} = q_{\bar{s}} - 1$ and $\widehat{q}_s = q_s$ for $s \neq \bar{s}$. Since $a = \widehat{\mu}(i)$ and $a' = \widehat{\mu}'(i)$ are different, the Rural Hospital Theorem (Roth, 1986) implies that a and a' are schools and fill their places in both $\widehat{\mu}$ and $\widehat{\mu}'$. Therefore, a and a' satisfy the same properties in μ and μ'. □

Lemma 4. *If $\mu'(i)P_i\mu(i)$, then i is part of a μ-improving cycle implementable by μ'.*

Proof. Let $G^* = (V^*, E^*)$ be a *multigraph* in which $V^* = S$ and there is a directed edge between s and s' *for each* student k such that $\mu(k) = s$ and $\mu'(k) = s'$, with $s \neq s'$. Since item (i) ensures that $|\mu(s)| = q_s$ and $|\mu'(s')| = q_{s'}$ for any s and s' such that $[s, s'] \in E^*$, the in-degree and the out-degree of each node in V^* coincide. Moreover, as schools $a = \mu(i)$ and $a' = \mu'(i)$ are different, there is at least one edge between a and a'.

As a consequence, there is a cycle of schools $(s_1, \ldots, s_r)$ in G^* such that $(s_1, s_2) = (a, a')$. Indeed, we can form this cycle departing from node a in direction to node a' and moving around the graph, because after enter a node $s_j \neq s_1$ from an edge, we are always able to exit from another edge. Since V^* is a finite set, we will eventually return to a. The construction of E^* allow us to associated to $(s_1, \ldots, s_r)$ a group of students $i_1, \ldots, i_r$ such that $i_1 = i$, $\mu(i_l) = s_l$ for all $l \in \{1, \ldots, r\}$, and $\mu'(i_l) = s_{l+1}$ for all $l \in \{1, \ldots, r\}$ [modulo r]. In particular, although the schools $s_1, \ldots, s_r$ are not necessarily different, we can assume that the students $i_1, \ldots, i_r$ are.

Since $\mu'(i_l) = s_{l+1}$, when $\mu'(i_l)P_{i_l}\mu(i_l)$ the stability of μ ensures that $j \succ_{s_{l+1}} i_l$ for all $j \in \mu(s_{l+1})$. Hence, the stability of μ' implies that any edge between s_{l+1} and s_{l+2} is associated to a student that improves we she moves from μ to μ'. In particular, $\mu'(i_{l+1})P_{i_{l+1}}\mu(i_{l+1})$. Therefore, $\mu'(i_1)P_{i_1}\mu(i_1)$ ensures that $\mu'(i_l)P_{i_l}\mu(i_l)$ for all $l \in \{1, \ldots, r\}$. We conclude that $(i_1, \ldots, i_r)$ is a μ-improving cycle implementable by μ' that includes the student $i_1 = i$. □

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